Polyphase Network Calculation using a Vector Analysis Method

Vector analysis leads to better design of 90° phase-shift networks.

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Introduction

It may seem difficult to understand the physical meaning of the polyphase networks that produce a 90° phase difference over a broad audio-frequency band. Many articles have been written on this subject since the polyphase network was first introduced to obtain a SSB signal.[1,2,3] However, no easily understood explanation has been published to date.

I intend to offer a simplified explanation of the method used to calculate polyphase networks using vector analysis based on basic trigonometry.

The amplitude response of a polyphase network is normally attenuated at mid-band frequencies when capacitors and resistors are selected in the traditional way. I propose a new system of selecting the capacitors and resistors that creates polyphase networks that have a flat amplitude response across the entire audio-frequency band.

The Vector Diagram of the Polyphase Network

A typical polyphase network is shown in Fig 1. When a push-pull audio signal is applied to the input terminals (1 to 4), four output signals having 90° phase differences appear at the output terminals at the right. (Capacitors and resistors in the same column have the same values.)

As shown in Fig 1, the same audio signal, E_i, is applied to input terminals 1 and 2, and an out-of-phase audio signal, $-E_i$, is applied to input terminals 3 and 4. The vectors at input terminals e_{01}, e_{02}, e_{03} and e_{04} are in a line, as shown in the vector diagram of Fig 2.

Now, let us consider the state of the output vectors in the first column, e_{11}, e_{12}, e_{13} and e_{14}, when nothing is connected to the output terminals of the first column. As e_{01} and e_{02} are the same and equal to E_i, it is evident that e_{11} is also same as E_i at all frequencies. Similarly, e_{13} is equal to $-E_i$. The vector e_{12} lies on the circle of Fig 2, which has a diameter of $2E_i$. At zero frequency, e_{12} coincides with e_{02} (or E_i), as the impedance of C1 is infinite. At a very high frequency, e_{12} coincides with e_{03} (or $-E_i$), as the impedance of C1 is negligibly small.

Since at this point nothing is connected to the column output terminals, the same current that flows through R1 flows through each associated C1.Vector e_r, the voltage appearing across R1, leads the voltage appearing at C1, vector e_c, by 90°. The amplitude of each vector is calculated

[1]Notes appear on page 15.

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as follows:

$$e_r = R_1 \cdot i$$

$$e_c = \frac{i}{j\omega C_1} \qquad \text{Eq 1}$$

where i is the current flowing through C_1 and R_1. The phase difference θ between e_{11} and e_{12} is:

$$\theta = 2\tan^{-1}\left(\frac{e_r}{e_c}\right) = 2\tan^{-1}(\omega C_1 \cdot R_1) = 2\tan^{-1}\left(\frac{\omega}{\omega_1}\right) \qquad \text{Eq 2}$$

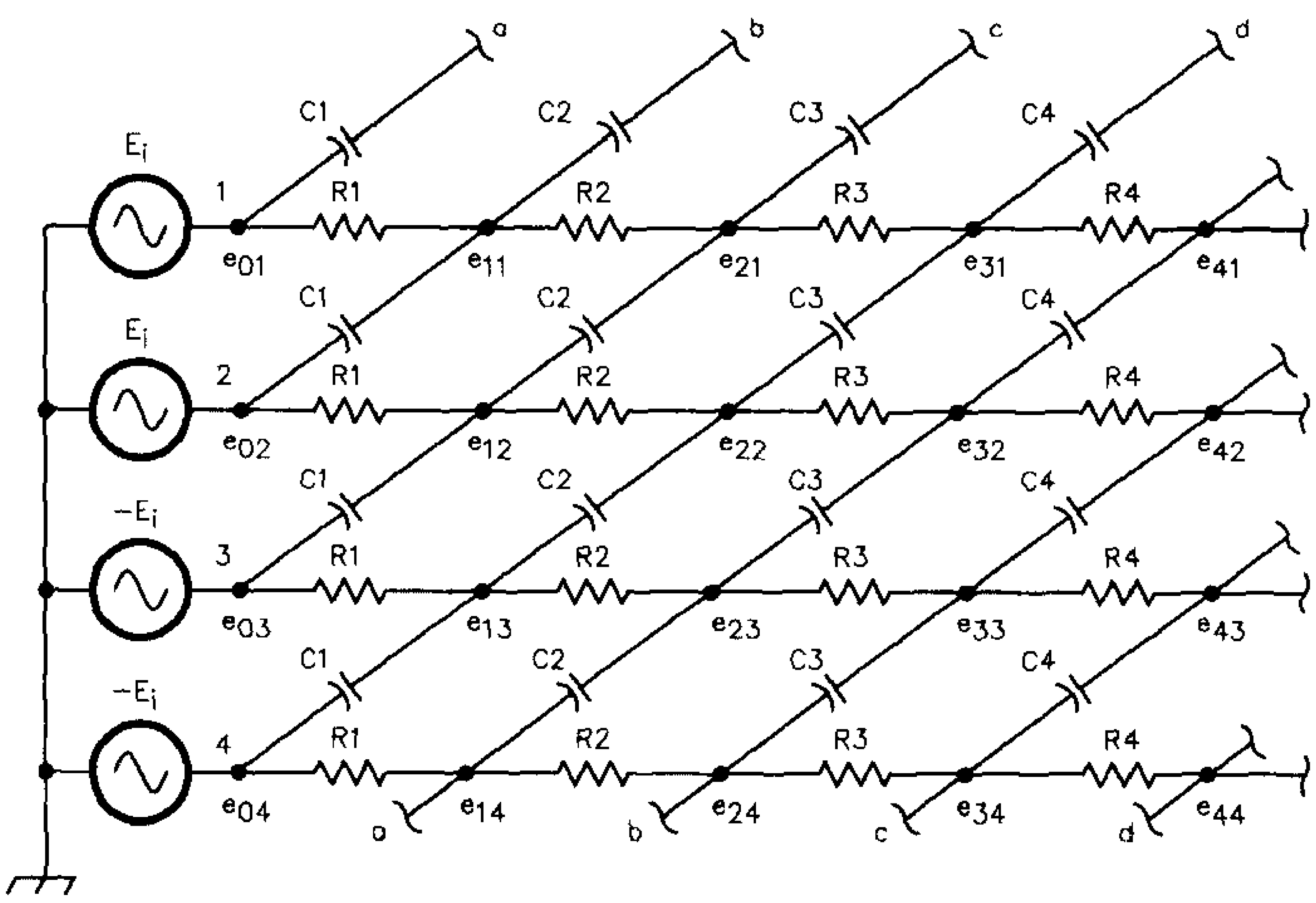

Fig 1—A typical polyphase network.

where

$$\omega_1 = \frac{1}{C_1 \cdot R_1} \qquad \text{Eq 3}$$

At a frequency of $0.5774/(2\pi C_1 \times R_1)$, for example, the angle between e_{11} and e_{12} is 60°, and at a frequency of $1/(2\pi C_1 \times R_1)$, it is 90°. Vector e_{14} also lies on the same circle but on the opposite side.

Now, let's connect the second column (C2 and R2). To make things simple, the impedance of the second column is set high compared to that of the first column. If the impedance of the second column is assumed to be high, vectors e_{11}, e_{12}, e_{13} and e_{14} in Fig 3 will be same as those of Fig 2.

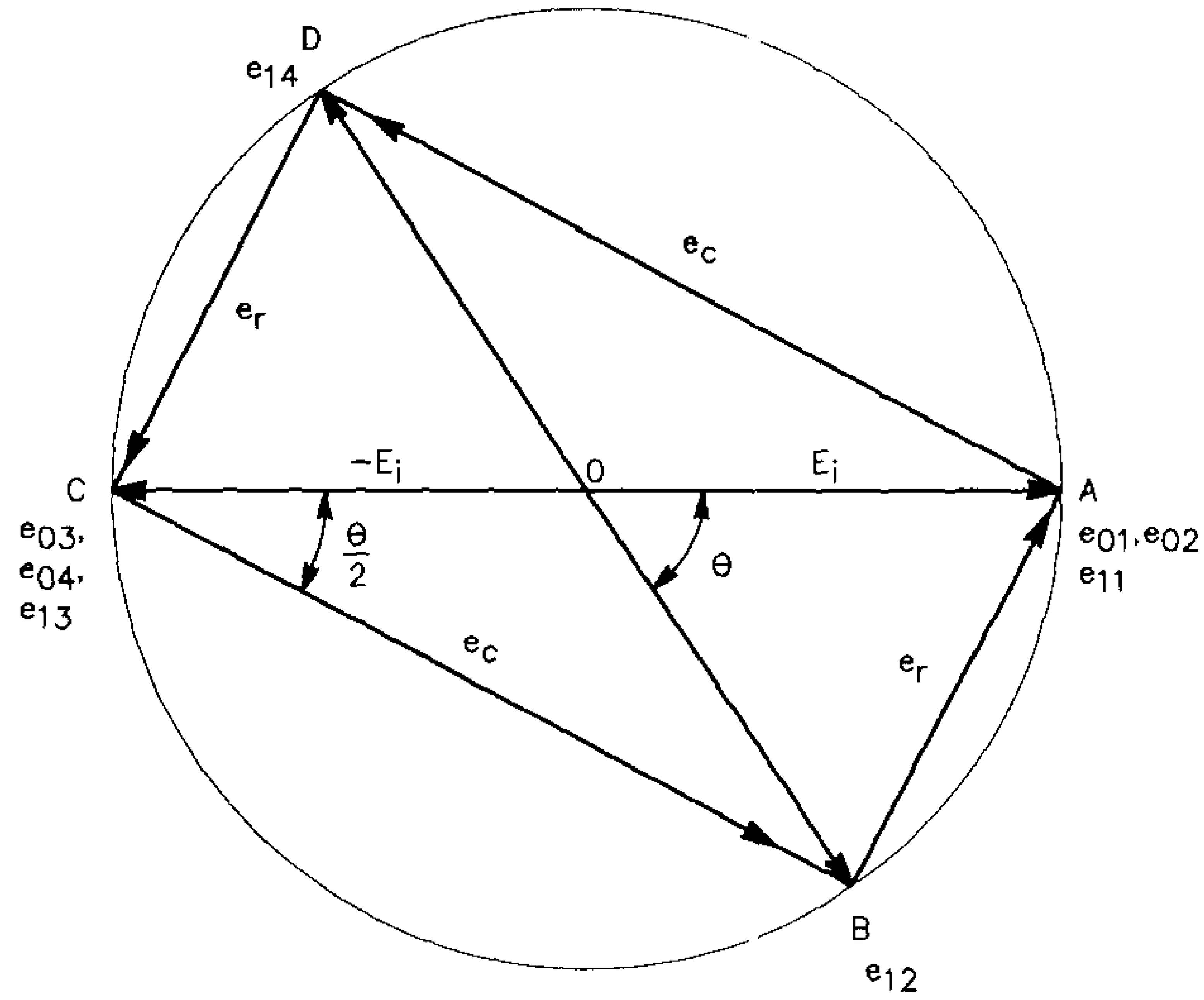

Fig 2—The vectors at the input and the output terminals of the first column.

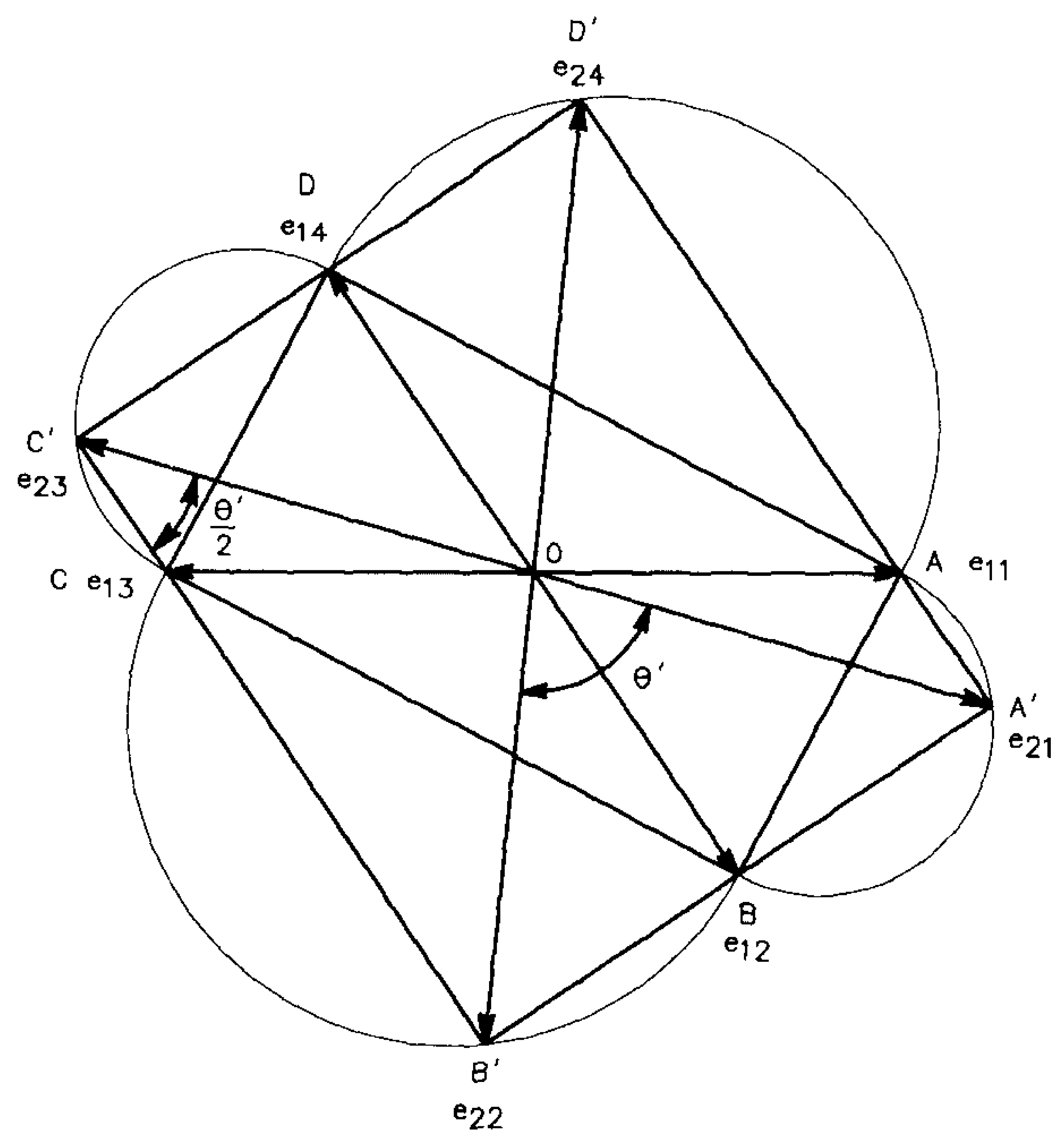

Fig 3—The vectors at the input and output terminals of the second column.

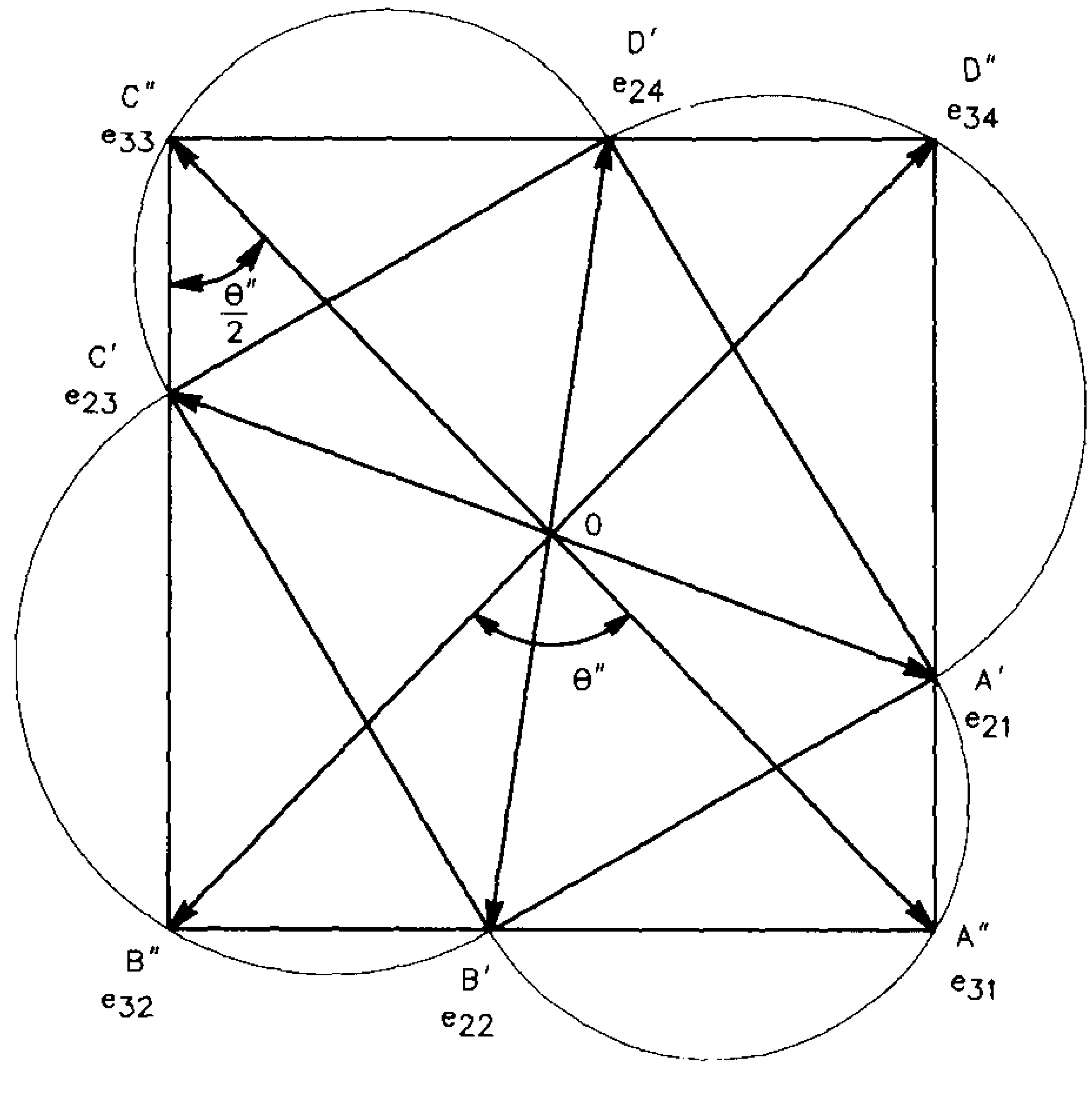

Fig 4—The vectors at the input and output terminals of the third column.

Vector e_{21} lies on the circle having a diameter AB, and vector e_{22} lies on the circle having a diameter BC. In Fig 3, it is easily shown that the angle between vectors e_{21} and e_{22} (that is the angle A′OB′) is twice the angle A′C′B′. Therefore, the angle between e_{21} and e_{22}, θ′ is:

$$\theta' = 2\tan^{-1}\left(\frac{A'B'}{B'C'}\right) \quad \text{Eq 4}$$

In the case of R1=R2 and C1=C2, at a frequency of $0.5774/(2\pi\, C_1{\times}R_1)$, the angle θ′ is 81.7868°, which is closer to 90° than the angle θ=60° at the output terminal of the first column.

Applying the same assumption of higher impedance, a third column can then be added. In Fig 4, the angle between vectors e_{31} and e_{32} is twice the angle A″ C″ B″ and can be obtained by:

$$\theta'' = 2\tan^{-1}\left(\frac{A''B''}{B''C''}\right) \quad \text{Eq 5}$$

Similarly, the angle at a frequency of $0.5574/(2\pi C_1{\times}R_1)$ is 87.7958° which is closer to 90° than that of the second column output.

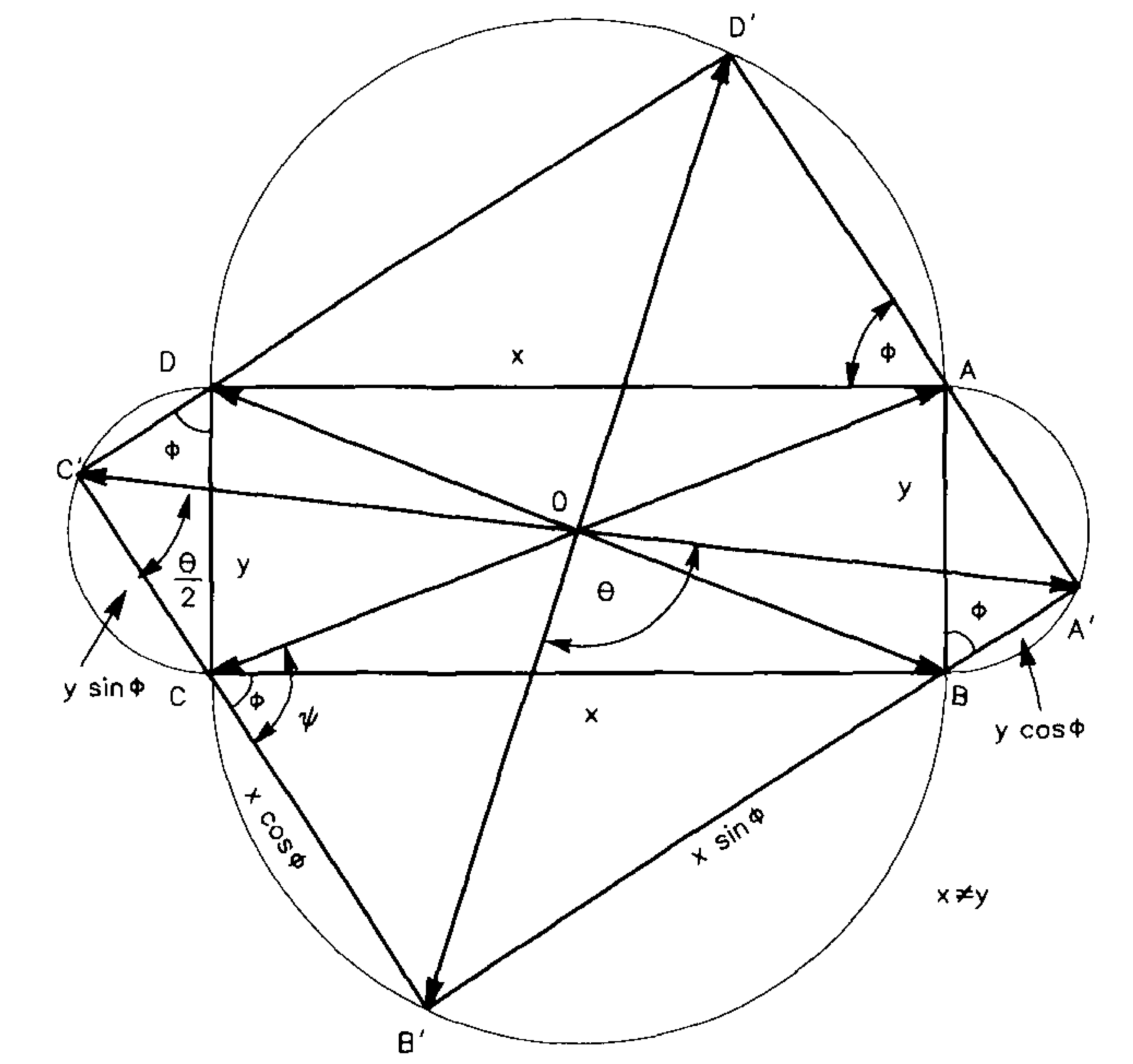

Fig 5

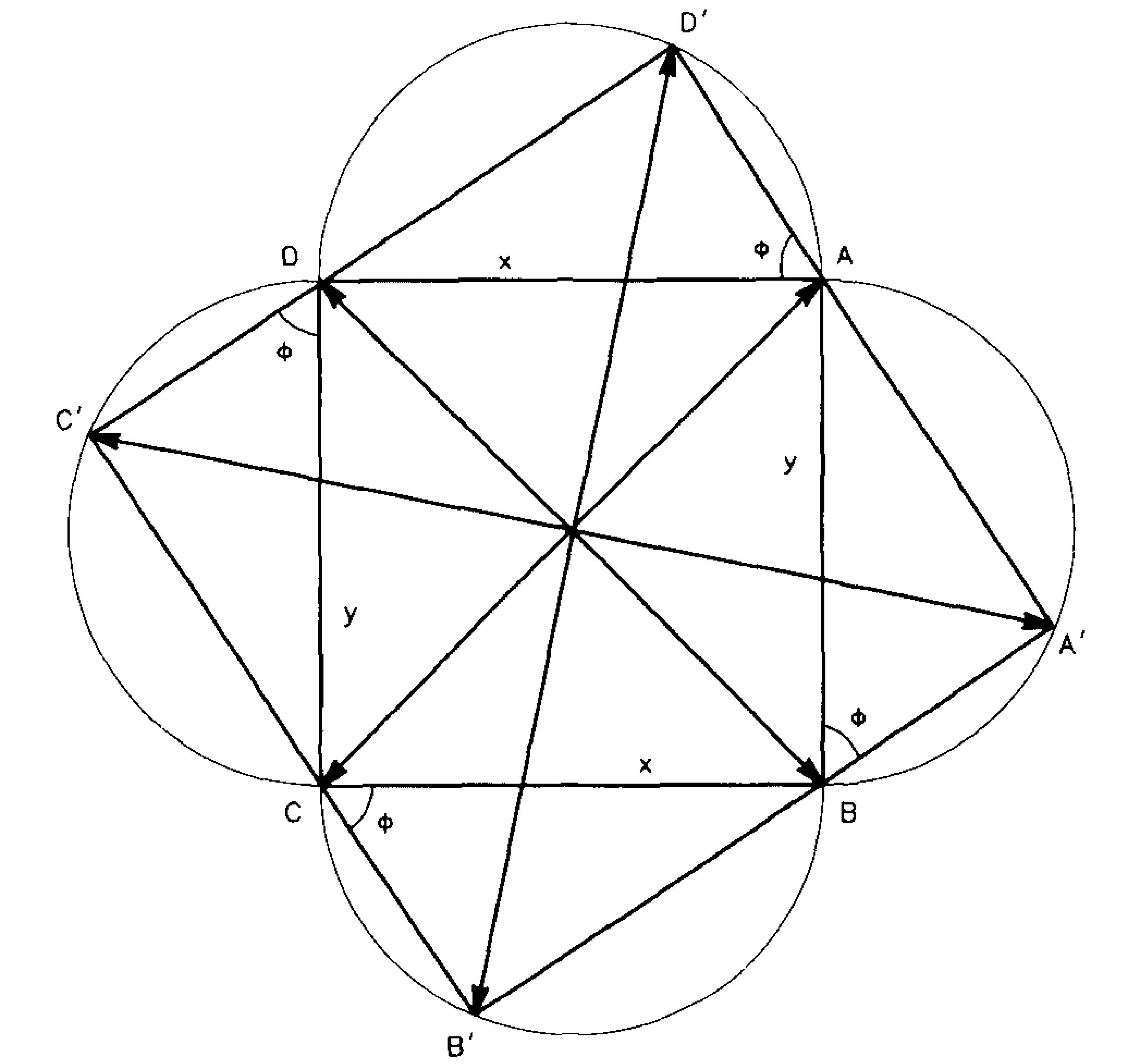

Fig 6

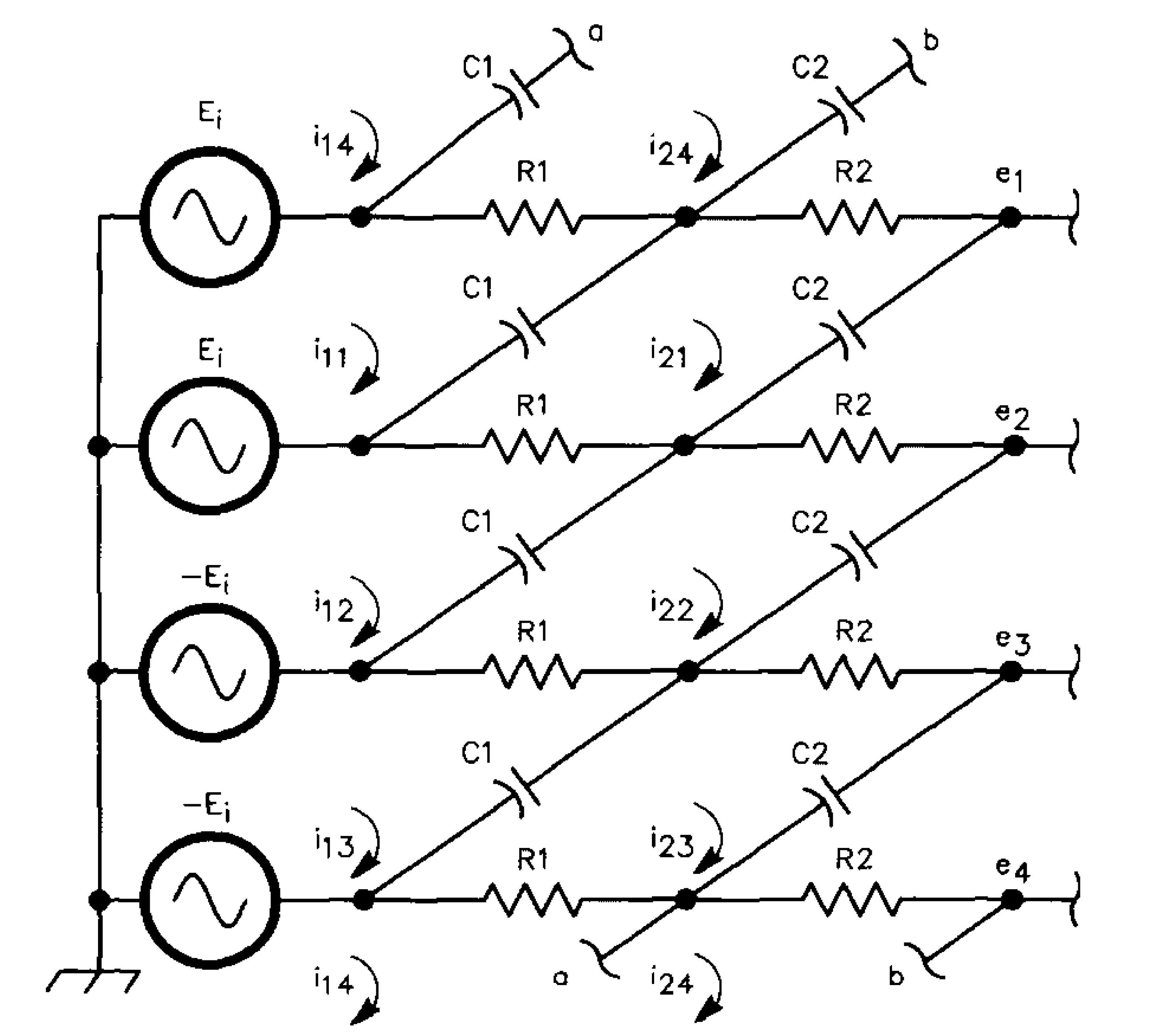

Fig 7—The circuit used to evaluate the impedance assumption.

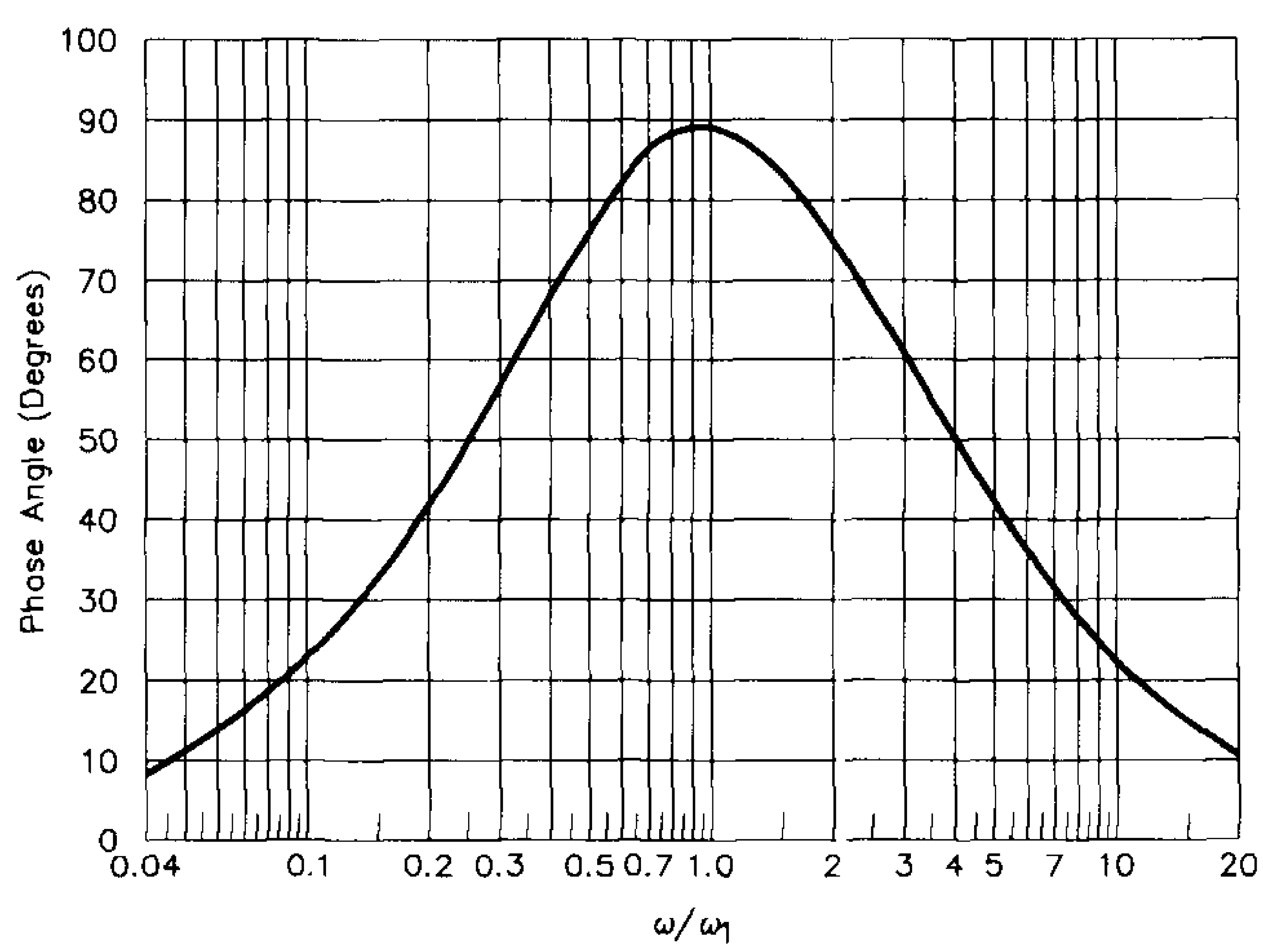

Fig 8—The phase difference between the vector e_1/E_i and e_2/E_i of Fig 7 ($\omega_1 = \omega_2$, any n).

As the audio signal passes through the columns, the phase angle between the output audio vectors gets closer and closer to 90°. Fig 5 explains this process. In this figure, A, B, C and D show the input vectors of a column and A′, B′, C′ and D′ show the output vectors of the same column. Applying the same assumption indicated above—the impedance of the next column is high enough compared to that of the preceding column:

$$A'B' = y\cos\phi + x\sin\phi$$
$$B'C' = x\cos\phi + y\sin\phi \qquad \text{Eq 6}$$

The ratio of the arms of the output rectangle is:

$$\frac{A'B'}{B'C'} = \frac{y\cos\phi + x\sin\phi}{x\cos\phi + y\sin\phi} \qquad \text{Eq 7}$$

As the ratio of the arms of the input rectangle is y/x, every time an audio signal passes through a column, the ratio changes as follows:

$$\frac{\dfrac{A'B'}{B'C'}}{\dfrac{AB}{BC}} = \frac{1+\dfrac{x}{y}\tan\phi}{1+\dfrac{y}{x}\tan\phi} \qquad \text{Eq 8}$$

If y/x is smaller than 1, the above ratio is larger than 1, and if y/x is more than 1, the above ratio would be less than 1. In all cases, the rectangles composed of four vectors become closer, forming a square as the vectors pass through columns.

When a column is added after the vectors achieve a 90° angle, the output vectors would still have the same 90° angle; the angles between the vectors are not affected. Fig 6 shows that if the input vectors become square, the output vectors are square. In this case, the result of the equation (Eq 8) becomes 1.

This is a very important result. In the polyphase network, if the vectors of an audio signal form a 90° angle, this 90° angle remains unchanged even if more columns are added afterwards, and this is the reason why the polyphase network is able to form a 90° phase shift over a very wide range of audio frequencies.

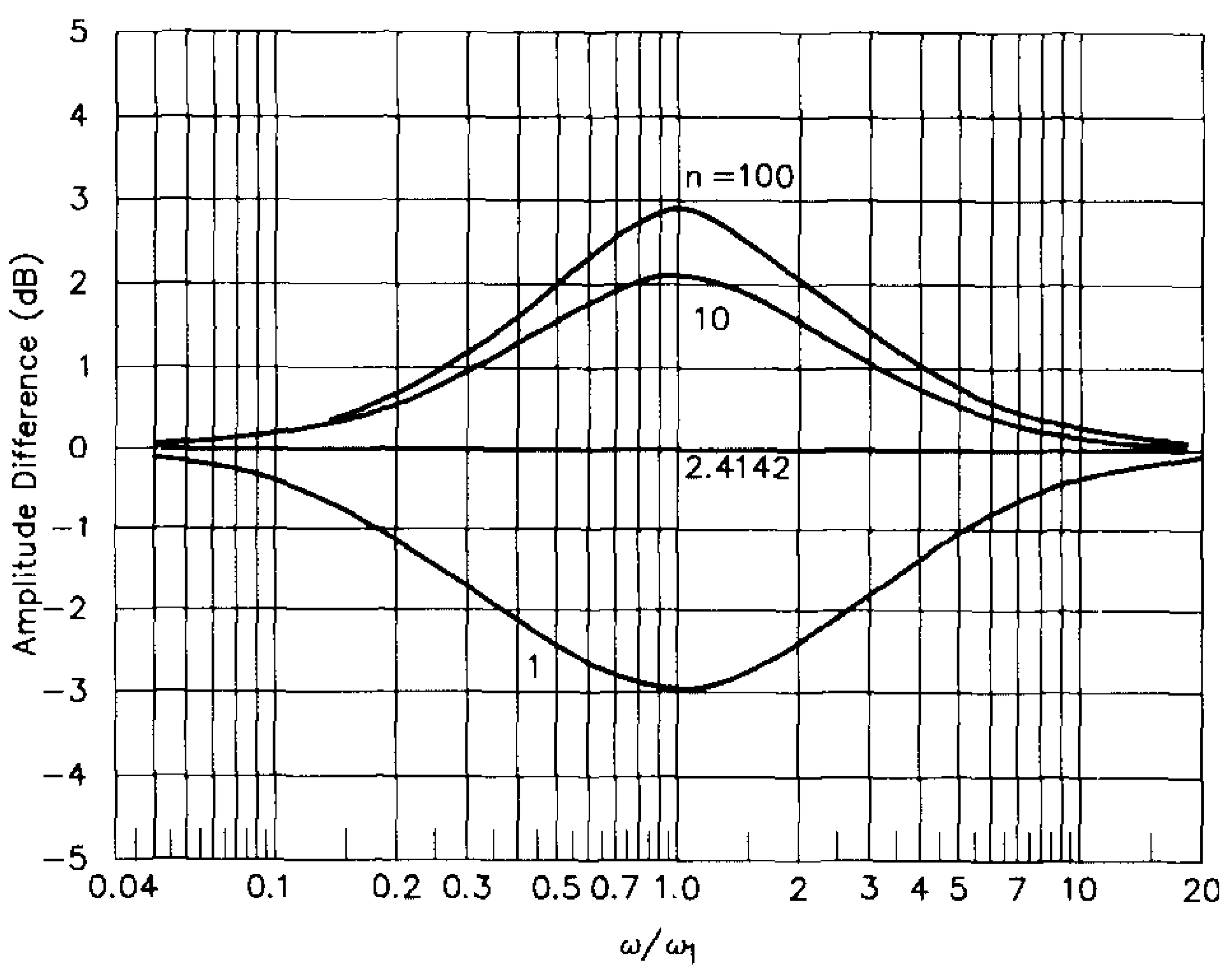

Fig 9—The difference in amplitude between the input and output terminals of Fig 7 ($\omega_1 = \omega_2$).

Evaluation of the Assumption

The above vector analysis is based on the assumption that the impedance of each column is high compared to that of the preceding column. If this assumption isn't true, what effect does the impedance of a column have on the vectors produced by the preceding columns?

The effect can be evaluated using the circuit shown in Fig 7, which is a two-column polyphase network. Applying Kirchhoff's law:

$$R_1(i_{11}-i_{24}) + \frac{1}{j\omega C_1}(i_{11}-i_{21}) = 0$$
$$R_1(i_{12}-i_{21}) + \frac{1}{j\omega C_1}(i_{12}-i_{22}) = 2E_i$$
$$R_1(i_{13}-i_{22}) + \frac{1}{j\omega C_1}(i_{13}-i_{23}) = 0$$
$$R_1(i_{14}-i_{23}) + \frac{1}{j\omega C_1}(i_{14}-i_{24}) = -2E_i$$
$$R_1(i_{21}-i_{12}) + \frac{1}{j\omega C_1}(i_{21}-i_{11}) + \left(R_2 + \frac{1}{j\omega C_2}\right)i_{21} = 0$$
$$R_1(i_{22}-i_{13}) + \frac{1}{j\omega C_1}(i_{22}-i_{12}) + \left(R_2 + \frac{1}{j\omega C_2}\right)i_{22} = 0 \qquad \text{Eq 9}$$
$$R_1(i_{23}-i_{14}) + \frac{1}{j\omega C_1}(i_{23}-i_{13}) + \left(R_2 + \frac{1}{j\omega C_2}\right)i_{23} = 0$$
$$R_1(i_{24}-i_{11}) + \frac{1}{j\omega C_1}(i_{24}-i_{14}) + \left(R_2 + \frac{1}{j\omega C_2}\right)i_{24} = 0$$

substituting:

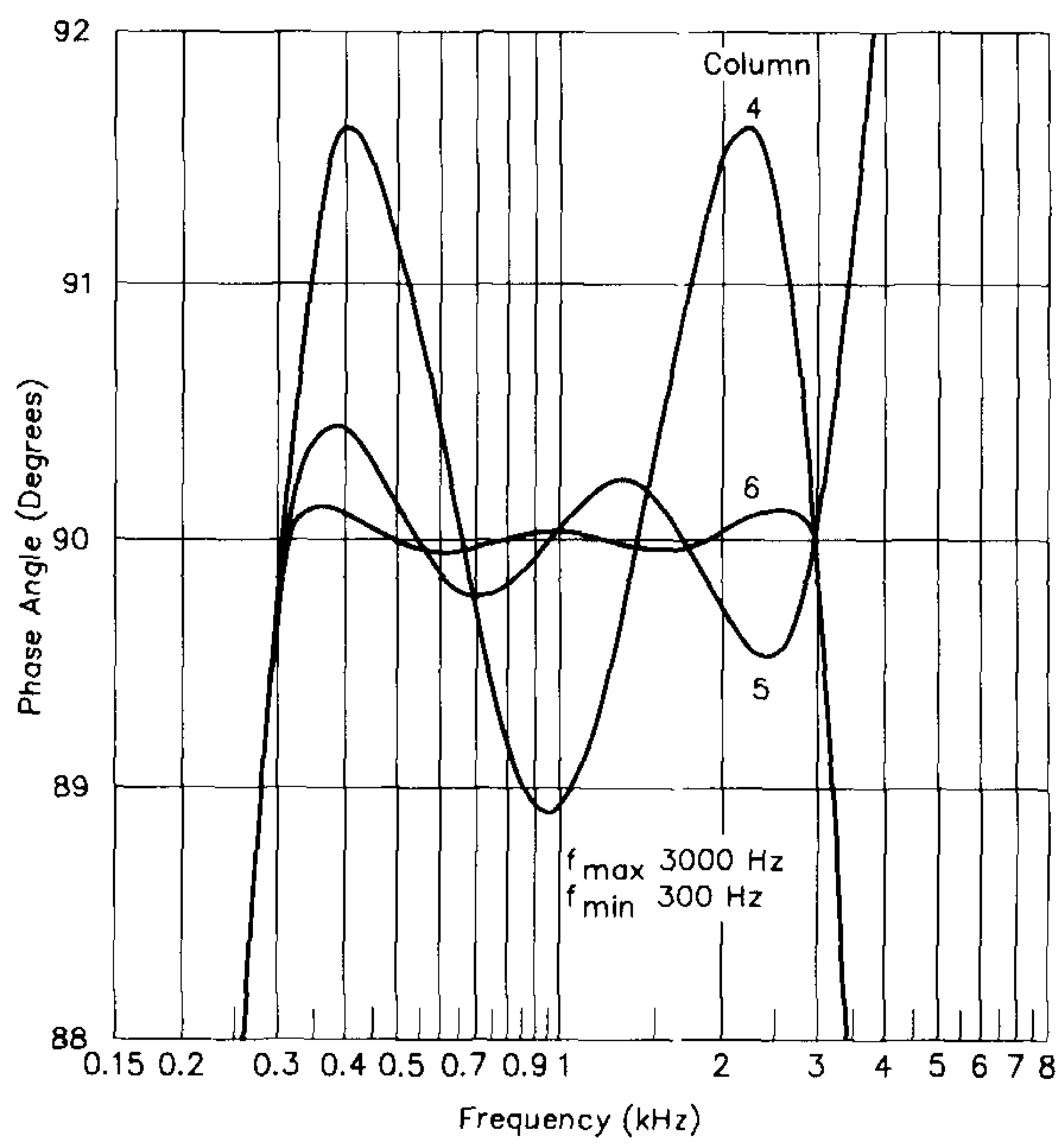

Fig 10—Phase difference of 4-, 5-, and 6-column polyphase networks.

$$\omega_1 = \frac{1}{C_1 \cdot R_1}, \omega_2 = \frac{1}{C_2 \cdot R_2}, n = \frac{R_2}{R_1} \qquad \text{Eq 10}$$

and showing the output vectors as e_1, e_2, e_3 and e_4, Eq 9 is solved as:

$$\frac{e_1}{E_i} = \frac{1 + \frac{\omega^2}{\omega_1 \cdot \omega_2} + j\left(\frac{\omega}{\omega_1} + \frac{\omega}{\omega_2}\right)}{1 - \frac{\omega^2}{\omega_1 \cdot \omega_2} + j\left(\frac{2\omega}{n \cdot \omega_2} + \frac{\omega}{\omega_1} + \frac{\omega}{\omega_2}\right)}$$

$$\frac{e_2}{E_i} = \frac{1 + \frac{\omega^2}{\omega_1 \cdot \omega_2} - j\left(\frac{\omega}{\omega_1} + \frac{\omega}{\omega_2}\right)}{1 - \frac{\omega^2}{\omega_1 \cdot \omega_2} + j\left(\frac{2\omega}{n \cdot \omega_2} + \frac{\omega}{\omega_1} + \frac{\omega}{\omega_2}\right)}$$

$$\frac{e_3}{E_i} = -\frac{1 + \frac{\omega^2}{\omega_1 \cdot \omega_2} + j\left(\frac{\omega}{\omega_1} + \frac{\omega}{\omega_2}\right)}{1 - \frac{\omega^2}{\omega_1 \cdot \omega_2} + j\left(\frac{2\omega}{n \cdot \omega_2} + \frac{\omega}{\omega_1} + \frac{\omega}{\omega_2}\right)} \qquad \text{Eq 11}$$

$$\frac{e_4}{E_i} = -\frac{1 + \frac{\omega^2}{\omega_1 \cdot \omega_2} - j\left(\frac{\omega}{\omega_1} + \frac{\omega}{\omega_2}\right)}{1 - \frac{\omega^2}{\omega_1 \cdot \omega_2} + j\left(\frac{2\omega}{n \cdot \omega_2} + \frac{\omega}{\omega_1} + \frac{\omega}{\omega_2}\right)}$$

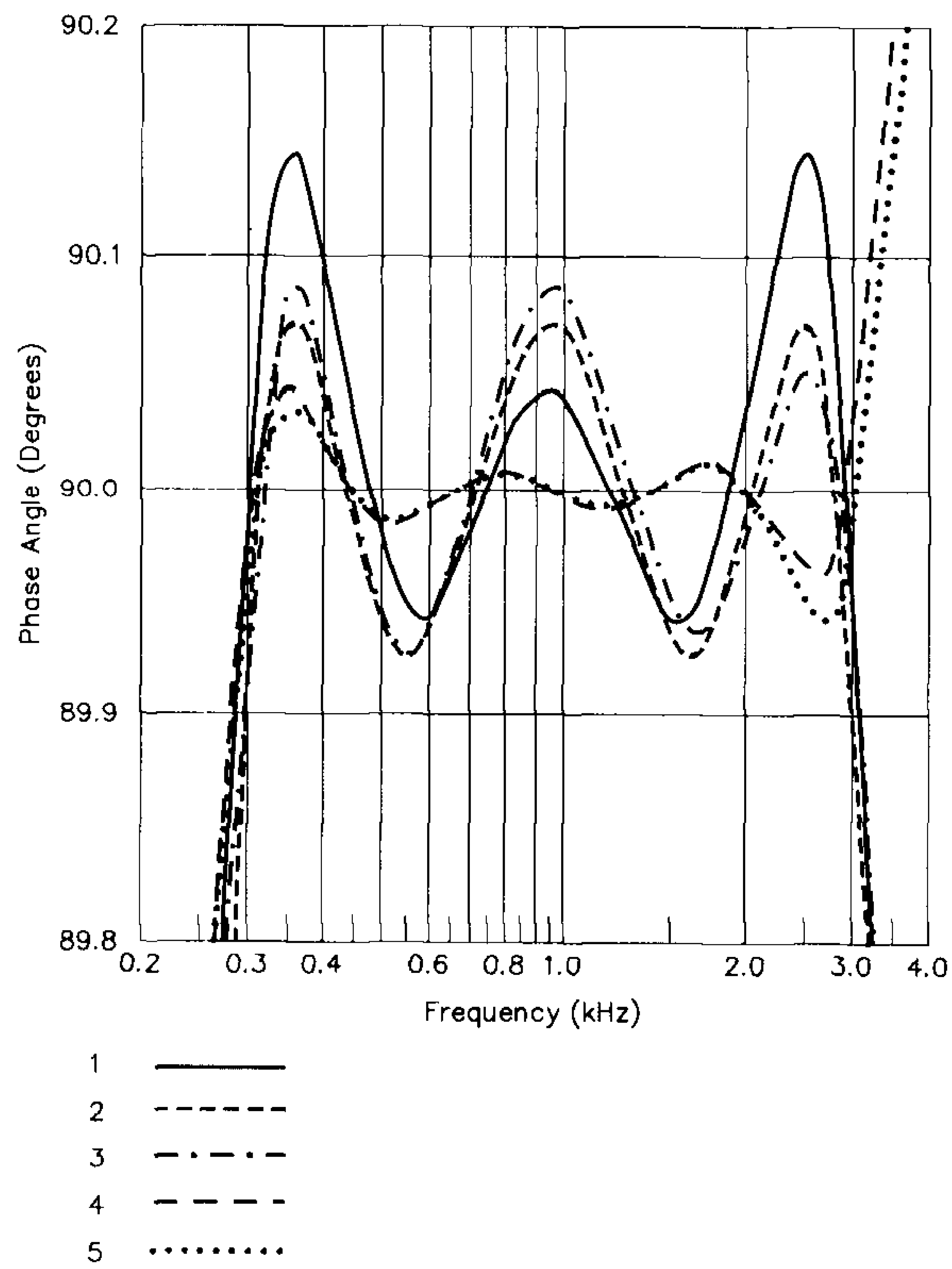

Fig 11—A comparison of standard networks with Chebychev networks.The networks are described in Table 1.

To make it simpler, suppose $\omega_1 = \omega_2$. The phase difference for this case between vectors e_1/E_i and e_2/E_i is shown in Fig 8. The amplitudes of the four vectors of Eq 8 are all identical and are shown in Fig 9. Fig 9 shows that the amplitudes of the output vectors are increased as n, the ratio of resistance of the next column to that of the preceding column, is increased. If n is 1, as has been the case so far, the output vector is decreased by 3 dB compared to the input vector at the center frequency. If $\omega_1 = \omega_2$, the output vector will have the same amplitude as that of the input vector when n is 2.414, or 1 plus $\sqrt{2}$. When n is increased more, the output vector becomes larger than the input vector—as much as 3 dB. It is interesting that although the amplitude is affected by n, the phase is *not* affected by n at all, as shown in Fig 8. This confirms that the assumption introduced here is generally acceptable.

I compared Fig 8 against values calculated using a vector method with the same assumption and found it to be true. Therefore, it can be concluded that, as far as the angles of the output vectors of polyphase networks are concerned, the vector analysis method based on basic trigonometry is applicable.

This analysis leads to another important fact about the ratio of the amplitudes of the output vectors to those of the input vectors. All the vector amplitudes of Eq 11 have the same value. Setting it to unity, the following equation is obtained:

$$n = \frac{m + 1 + \sqrt{m^2 + 6m + 1}}{2m} \qquad \text{Eq 12}$$

where

$$m = \frac{\omega_2}{\omega_1} \qquad \text{Eq 13}$$

This means that the resistance of each column must be n times of that of the preceding column, and that value changes according to the ratio of the time constants of the the two columns.

Calculation of the Angle between Output Vectors

Referring to Fig 5 and Eq 6, the angle between output vectors and the phase-shift value between input and output vectors are obtained by a series of calculations using the following equations:

$$f(p) = \frac{1}{2\pi C_p \cdot R_p}$$

$$S(p) = \tan^{-1}\left\{\frac{f}{f(p)}\right\} = \phi$$

$$X(p) = X(p-1) \cdot \cos\{S(p)\} + Y(p-1) \cdot \sin\{S(p)\}$$

$$Y(p) = Y(p-1) \cdot \cos\{S(p)\} + X(p-1) \cdot \sin\{S(p)\}$$

$$H(p) = 2\tan^{-1}\left\{\frac{Y(p)}{X(p)}\right\} = \theta$$

$$K(p) = \tan^{-1}\left\{\frac{Y(p)}{X(p)}\right\} = \frac{\theta}{2} \qquad \text{Eq 14}$$

$$L(p) = \tan^{-1}\left[\frac{Y(p)}{X(p-1) \cdot \cos\{S(p)\} - Y(p-1) \cdot \sin\{S(p)\}}\right] = \Psi$$

$$M(p) = L(p) - K(p)$$

where

p is the number of columns, from 1 to c, and p–1 means the number of the preceding column.

C_p and R_p are the capacitance and resistance of the pth column.

$H(p)$ is the angle between the pth column output vectors of the 1st row and 2nd row.

$M(p)$ is the phase shift between the pth and (p-1)th columns of the first row.

$X(0)$ and I(0) are the input audio vector components, and $X(0)=1$ and $Y(0)=0$ can be used as initial conditions.

Examples of calculated values for 4-, 5- and 6-column polyphase networks are shown in Fig 10. In this calculation, the node frequencies of each column, which are equal to $1/(2\pi C\times R)$, are determined by the following equations:

$$f_p = f_{min} \cdot \left(\frac{f_{max}}{f_{min}} \right)^{\left(\frac{P-1}{C-1} \right)} \qquad \text{Eq 15}$$

where

f_{min} is the lowest frequency of the audio pass band and is f_1.

f_{max} is the highest frequency of the audio pass band and is f_c.

The resistance used in each column is increased over that of the preceding column by n (Eq 12).The column capacitance is obtained by:

$$C_p = \frac{1}{2\pi f_p \cdot R_p} \qquad \text{Eq 16}$$

In this case, the node frequencies makes a geometrical progression. A beautiful Chebychev curve may be obtained if the node frequencies are selected carefully. Fig 11 shows a comparison of a 6-column network designed using the equations mentioned above (curve 1) with a Chebychev 6-column network by W9CF (curve 2) (see Note 2). The node

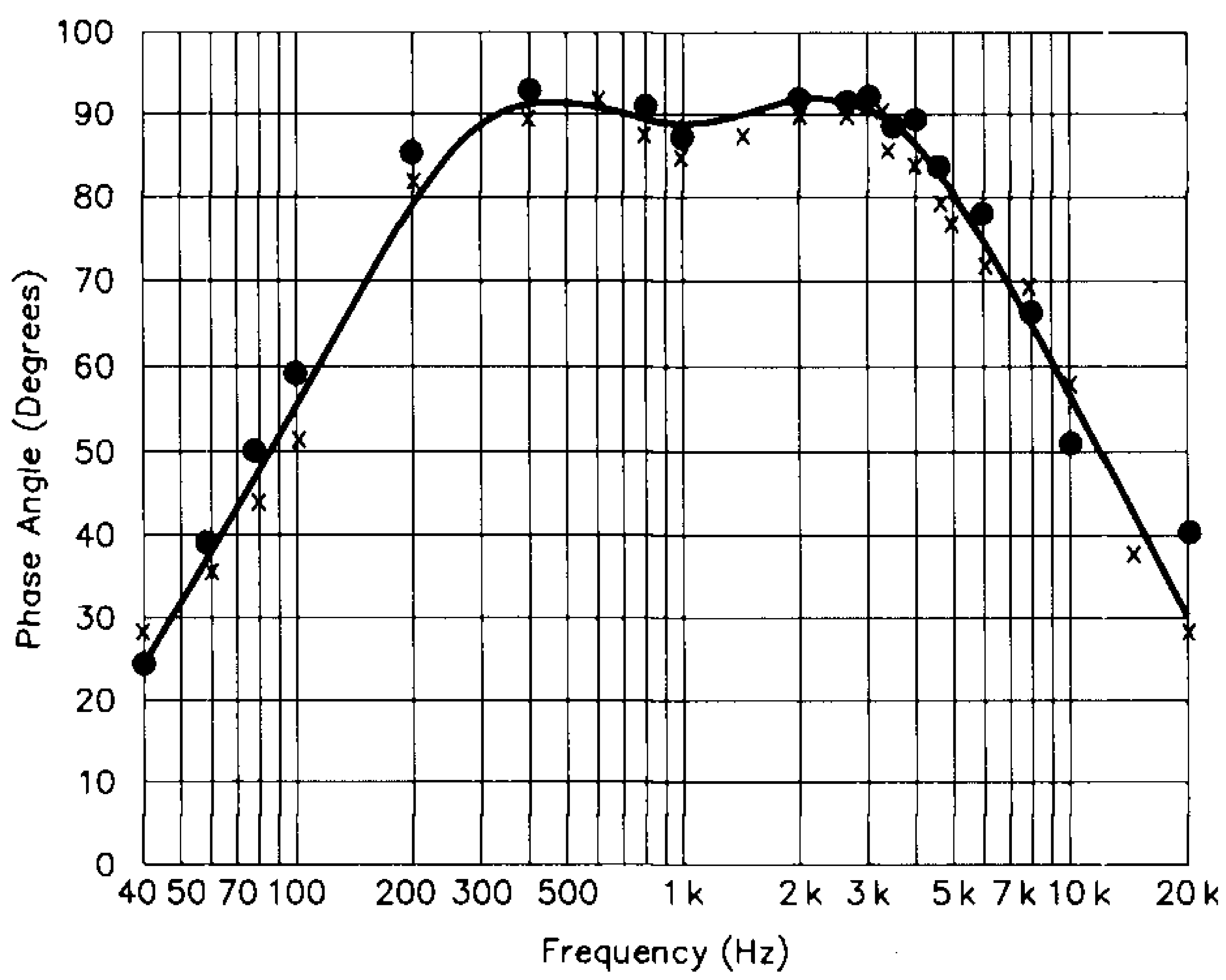

Fig 12—The phase angle of two example 4-column networks.
• Resistance value obtained using Eq 12.
x Constant resistance value.

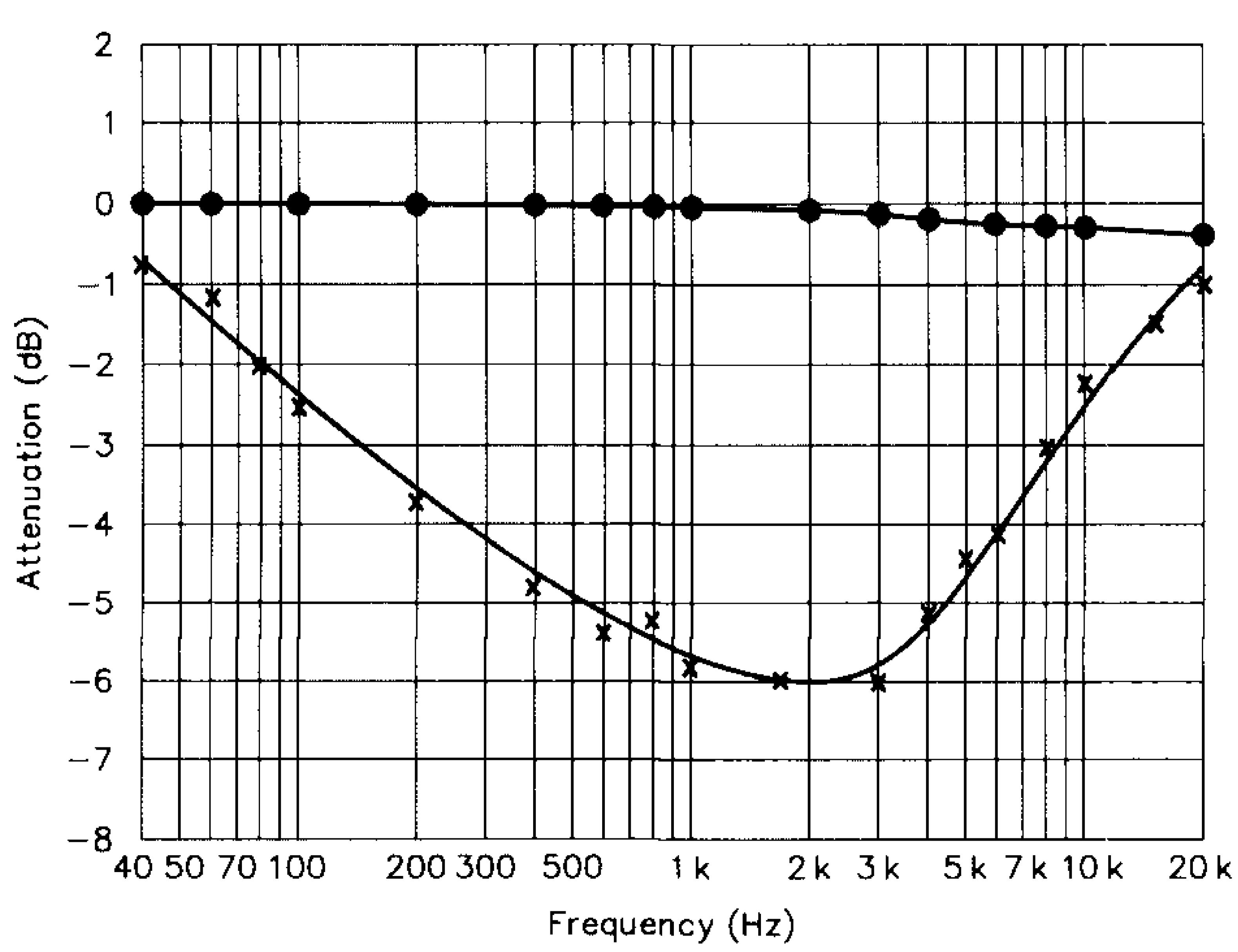

Fig 13—The attenuation of the 4-column networks.
• Resistance value obtained using Eq 12.
x Constant resistance value.

Table 1

Data of the networks (f in Hz, C in µF, R in kΩ)

Mark	1	2	3	4	5
f_1	300.0	314.2	312.1	300.0	312.1
f_2	475.5	435.5	442.1	440.3	442.1
f_3	753.6	720.3	723.4	646.3	658.2
f_4	1194.3	1249.5	1326	948.7	970.5
f_5	1892.9	2066.8	2122	1392.5	1463
f_6	3000.0	2864.5	2842	2043.9	2010
f_7	_	_	_	3000.0	3121
R_1	10.000	10.000	10	10.00	10
R_2	19.539	10.000	10	20.188	20
R_3	38.176	10.000	10	40.754	39
R_4	74.592	10.000	10	82.273	82
R_5	125.744	10.000	10	166.09	160
R_6	284.767	10.000	10	335.30	330
R_7	_	_	_	676.89	680
C_1	0.05305	0.05065	0.051	0.05305	0.051
C_2	0.01713	0.03655	0.036	0.01791	0.018
C_3	0.005532	0.02210	0.022	0.006042	0.0062
C_4	0.001787	0.01274	0.012	0.002039	0.0020
C_5	0.0005769	0.007701	0.0075	0.0006881	0.00068
C_6	0.0001863	0.005556	0.0056	0.000232	0.00024
C_7	_	_	_	0.00007838	0.000075

Table 2

f_p in Hz, C_p in µF, R_p in kΩ

Number of Column	First Network f_p	C_p	R_p	Second Network f_p	C_p	R_p
1	312.1	0.0068	75	294.7	0.018	30
2	643	0.0033	75	664	0.0047	52
3	1415	0.0015	75	1457	0.0012	91
4	3121	0.00068	75	3215	0.00033	150

frequencies and component values are shown in Table 1 mark 1 and mark 2.

The angle deviations from 90° in the audio passband are 0.07 degrees better in the case of the 6-column Chebychev than for the geometrical progression circuit described here, for a ratio between f_{max} and f_{min} of 10. It is more practical, however, to choose node frequencies determined by using capacitors and resistors of standard values that are readily available. An example is shown in Table 1 mark 3. Curve 3 in Fig 11 shows a case where standard value capacitors and resistors are used in a Chebychev network. When the number of columns is increased by one —to 7 columns instead of 6—and the exact-value C_s and R_s are used (mark 4 Table 1), the improvement in deviation is 0.1° (curve 4 of Fig 11). When standard-value C_s and R_s are used in a 7-column network (mark 5 Table 1), the angle deviations from 90° are slightly better (curve 5 of Fig 11) than the value of the ideal Chebychev 6-column network (curve 2 of Fig 11).

This means that it may be more economical to increase the number of columns than to try to procure uncommon values of resistors and capacitors.

An Actual Example and Conclusions

A pair of polyphase networks of four columns each were prepared and measured. The first network was constructed using the current basic design in which the values of all resistances are equal. The second network was designed using the technique proposed here, in which the values of resistance are increased according to Eq 12. The component data are shown in Table 2. The measured circuit responses are shown in Figs 12 and 13.

Fig 12 shows the angles between the output vectors of each network. The measured values here show no significant phase difference between the two networks. On the other hand, Fig 13 shows the attenuation of the networks, and it is quite clear that the network using increasing resistance values displays better frequency response.

In conclusion, I've explained why polyphase networks can produce a wide-band 90° phase difference and have shown a method that improves the amplitude response of such networks. Although the analysis is based on certain assumptions, the measurements taken from the actual sample network confirm that this theory can be valuable for practical network design.

Notes

[1]Gingell, M. J., "Single Sideband Modulation using Sequence Asymmetric Polyphase Networks," *Electrical Communication*, 1973, Volume 48, Number 1 and 2, pp 21-25.

[2]Gschwindt, A. (HA5WH), "Some reflections on the four-way phasing method," *Radio Communication*, 1976, January, pp 28-33.

[3]Schmidt, Kevin (W9CF), "Phase-Shift Network Analysis and Optimization," *QEX*, 1994, April, pp 17-23. □□